THE QUALITY ENGINEER PRIMER

MOST PEOPLE WOULD RATHER LIVE WITH A PROBLEM THEY CAN'T SOLVE, THAN ACCEPT A SOLUTION THEY CAN'T UNDERSTAND.

R. E. D. WOOLSEY AND H. S. SWANSON

Advanced Statistics

Advanced Statistics is presented in the following topic areas:

- **Statistical decision making**
- **Analysis of variance (ANOVA)**
- **Relationships between variables**
- **Design and analysis of experiments**

Statistical Decision Making

Statistical Decision Making is presented in the following topic areas:

- **Point estimates**
- **Confidence intervals**
- **Hypothesis testing**
- **Paired-comparison tests**
- **Goodness-of-fit tests**
- **Contingency tables**

Statistical Inference

The objective of statistical inference is to draw conclusions about population characteristics based on the information contained in a sample. The steps involved in statistical inference are:

- **Precisely define the problem objective**
- **Formulate a null hypothesis and an alternate hypothesis**
- **Decide if the problem will be evaluated by a one-tail or two-tail test**
- **Select a test distribution and a critical value for the test statistic**
- **Calculate a test statistic value from the sample information**
- **Make a inference by comparing the calculated and the critical values**
- **Report the findings**

Note that the procedure for statistical inference is very similar to the hypothesis testing procedure presented later in this Section.

Point Estimate for Population Mean

In analyzing sample values to arrive at population probabilities, two major estimators are used: point estimates and confidence intervals.

A point estimate of the population mean, μ, is the sample mean, $\bar{X}$.

$$\mu \approx \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Example 11.1 Given the following tensile strength readings from 4 piano wire segments: 28.7, 27.9, 29.2, and 26.5 psi, calculate the point estimation of the population mean.

$$\mu \approx \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{28.7 + 27.9 + 29.2 + 26.5}{4} = 28.08 \text{ psi}$$

28.08 psi is the point estimate for the population mean.

Point Estimate for Population Variance

The sample variance, s^2, is the best point estimate of the population variance, σ^2. The sample standard deviation, s, is the best point estimate of the population standard deviation, σ.

$$s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1} \qquad \sigma^2 = \frac{\sum_{i=1}^{n} (X_i - \mu)^2}{N}$$

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n - 1}} \qquad \sigma = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \mu)^2}{N}}$$

The determination of s and s^2 was covered previously in Primer Section IX.

Confidence Interval for the Mean

Continuous Data - σ Known

The confidence interval (CI) or interval estimate of the population mean, μ, when the population standard deviation, σ, is known, is calculated using the sample mean, $\bar{X}$, the population standard deviation, σ, the sample size, n, and the normal distribution.

$$\bar{X} - Z_{\alpha/2}\frac{\sigma_X}{\sqrt{n}} \le \mu \le \bar{X} + Z_{\alpha/2}\frac{\sigma_X}{\sqrt{n}}$$

From sample data one can calculate the interval within which the population mean, μ, is predicted to fall. Confidence intervals are always estimated for population parameters. A confidence interval is a two-tail event and requires critical values based on an alpha/2 risk in each tail. The central limit theorem term, $\sigma/\sqrt{n}$, is necessary because the confidence interval is for a population mean and not individual values.

Example 11.2: The average of 100 samples is 18 with a population standard deviation of 6. Calculate the 95% confidence interval for the population mean.

$$\bar{X} - Z_{\alpha/2}\frac{\sigma_X}{\sqrt{n}} \le \mu \le \bar{X} + Z_{\alpha/2}\frac{\sigma_X}{\sqrt{n}}$$

$$18 - 1.96\frac{6}{\sqrt{100}} \le \mu \le 18 + 1.96\frac{6}{\sqrt{100}}$$

$$16.82 \le \mu \le 19.18$$

Continuous Data - σ Unknown

The confidence interval of the population mean, μ, when the population standard deviation, σ, is unknown, is calculated using the sample mean, $\bar{X}$, the sample standard deviation, s, the sample size, n, and the t distribution. If a relatively small sample is used, e.g. n < 30, then the t distribution must be used. When the sample size is large, e.g. n > 30, the Z distribution may be used in place of the t distribution.

$$\bar{X} - t_{\alpha/2,\, n-1}\frac{s}{\sqrt{n}} \le \mu \le \bar{X} + t_{\alpha/2,\, n-1}\frac{s}{\sqrt{n}}$$

Confidence Interval for the Mean (Continued)

Continuous Data - σ Unknown (Continued)

Example 11.3: The average of 25 samples is 18 with a sample standard deviation of 6. Calculate the 95% confidence interval for the population mean.

$$\bar{X} - t_{\alpha/2,\, n-1}\frac{s}{\sqrt{n}} \le \mu \le \bar{X} + t_{\alpha/2,\, n-1}\frac{s}{\sqrt{n}}$$

$$18 - 2.064\frac{6}{\sqrt{25}} \le \mu \le 18 + 2.064\frac{6}{\sqrt{25}}$$

$$15.52 \le \mu \le 20.48$$

Confidence Interval for Proportion

For large sample sizes, with np and n(1-p) greater than or equal to 5, the binomial distribution can be approximated by the normal distribution to calculate a confidence interval for population proportion. The following formula is used:

$$p_s - Z_{\alpha/2}\sqrt{\frac{p_s(1 - p_s)}{n}} \le p \le p_s + Z_{\alpha/2}\sqrt{\frac{p_s(1 - p_s)}{n}}$$

Where: p_s = sample proportion, p = population proportion, n = sample size

Note that other confidence interval formulas exist. These include percent nonconforming, Poisson distribution data, and very small sample size data.

Example 11.4: If 16 defectives were found in a sample size of 200 units, calculate the 90 % confidence interval for the proportion.

$$p_s - Z_{\alpha/2}\sqrt{\frac{p_s(1 - p_s)}{n}} \le p \le p_s + Z_{\alpha/2}\sqrt{\frac{p_s(1 - p_s)}{n}}$$

$$p_s = \frac{x}{n} = \frac{16}{200} = 0.08$$

$$0.08 - 1.645\sqrt{\frac{0.08(1 - 0.08)}{200}} \le p \le 0.08 + 1.645\sqrt{\frac{0.08(1 - 0.08)}{200}}$$

$$0.048 \le p \le 0.112$$

Confidence Interval for Variance

The confidence interval or interval estimate for the population variance, σ^2, is given by:

$$\frac{(n-1)s_X^2}{X^2_{\alpha/2,\, n-1}} \le \sigma^2 \le \frac{(n-1)s_X^2}{X^2_{1-\alpha/2,\, n-1}}$$

Where: s^2 **= sample variance**
n **= sample size**
$n - 1$ **= degrees of freedom**

The confidence interval for the mean were symmetrical about the average. This is not true for the variance, since it is based on the chi-square distribution.

Example 11.5: The sample variance for a set of 25 samples was found to be 36. Calculate the 90% confidence interval for the population variance.

$$\frac{(n-1)s_X^2}{X^2_{\alpha/2,\, n-1}} \le \sigma^2 \le \frac{(n-1)s_X^2}{X^2_{1-\alpha/2,\, n-1}}$$

$$\frac{(25-1)36}{36.42} \le \sigma^2 \le \frac{(25-1)36}{13.85}$$

$$23.72 \le \sigma^2 \le 62.38$$

Confidence Interval for Standard Deviation

The confidence interval for the population standard deviation, σ, is given by:

$$\sqrt{\frac{(n-1)s_X^2}{X^2_{\alpha/2,\, n-1}}} \le \sigma \le \sqrt{\frac{(n-1)s_X^2}{X^2_{1-\alpha/2,\, n-1}}}$$

Hypothesis Testing

Hypothesis testing is a type of statistical inference in which a null hypothesis and alternative hypothesis are stated. The null hypothesis is a statement about the value of a population parameter such as the mean, and must contain the condition of equality. The alternative hypothesis is a statement that must be true if the null hypothesis is false. <u>A null hypothesis can only be rejected, or fail to be rejected, it cannot be accepted because of a lack of evidence to reject it.</u> (Triola, 1994)[9]

As an example of hypothesis tests for a population mean, there are only three possible forms, where μ is the population mean and μ_0 is a specified value:

$H_0: \mu = \mu_0$	$H_0: \mu \leq \mu_0$	$H_0: \mu \geq \mu_0$
$H_1: \mu \neq \mu_0$	$H_1: \mu > \mu_0$	$H_1: \mu < \mu_0$

In order to test a null hypothesis, a test calculation must be made from sample information. This calculated value is called a test statistic and is compared to an appropriate critical value. A decision is then made to reject or to fail to reject the null hypothesis. The steps of hypothesis testing are:

- **State the null and alternative hypothesis**
- **Specify the level of significance, α**
- **Determine the critical values separating the reject and nonrejection areas**
- **Determine the sampling distribution and test statistic**
- **Determine if the test statistic is in the reject or nonrejection area**
- **Conclude if the null hypothesis is rejected or failed to be rejected**
- **State the statistical decision in terms of the original problem**

Additional information on statistical inference and hypothesis testing for specific tests is provided later in this Section.

Types of Errors

When formulating a conclusion regarding a population based on observations from a small sample, two types of errors are possible:

- Type I error: This error results when the null hypothesis is rejected when it is, in fact, true. The probability of making a type I error is called α (alpha) and is commonly referred to as the producer's risk (in sampling). Examples are: incoming products are good but called bad; a process change yields no improvement but is thought to be different.

- Type II error: This error results when the null hypothesis is not rejected when it should be rejected. This error is called the consumer's risk (in sampling) and is denoted by the symbol β (beta). Examples are: incoming products are bad, but called good; an adverse process change has occurred but is thought to be no different.

The degree of risk (α) is normally chosen by the concerned parties (α is often taken as 5%) in arriving at the critical value of the test statistic, the assumption is that a small value for α is desirable. Unfortunately, a small α risk increases the β risk. For a fixed sample size, α and β are inversely related. Increasing the sample size can reduce both the α and β risks.

The types of errors are shown in Figure 11.1 below:

		Null Hypothesis True	Null Hypothesis False
The Decision Made	Fail to Reject H_0	$p = 1 - \alpha$ Correct Decision	$p = \beta$ Type II Error
	Reject H_0	$p = \alpha$ Type I Error	$p = 1 - \beta$ Correct Decision

Figure 11.1 Error Matrix

One-Tail Test vs. Two-Tail Test

Any test of hypothesis has a risk associated with it. One is generally concerned with the α risk (a type I error which rejects the null hypothesis when it is true). The level of this α risk determines the level of confidence (1 - α) that is placed in the conclusion. This risk factor is used to determine the critical value of the test statistic which is compared to a calculated value.

One-Tail Test

If a null hypothesis is established to test whether a sample value is smaller or larger than a population value, then the entire α risk is placed on one end of a distribution curve. This constitutes a one tail-test.

- **A study was conducted to determine if the mean battery life produced by a new method is greater than the present battery life of 35 hours. In this case, the entire α risk will be placed on the right-tail of the existing life distribution curve.**

H_0: new $\leq$ to present **H_1: new > present**

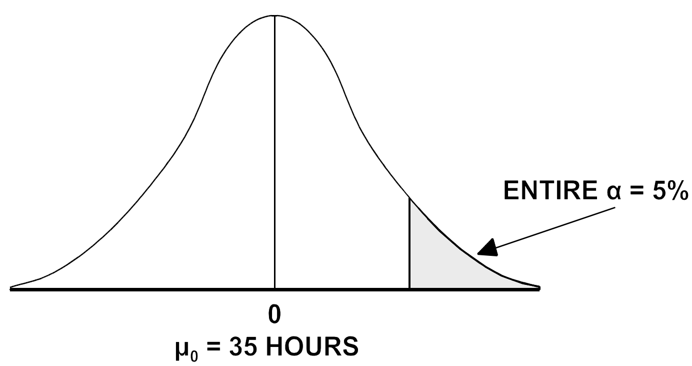


Figure 11.2 Determine if the true mean is within the α critical region.

- **A chemist is studying the vitamin levels in a brand of cereal to determine if the process level has fallen below 20% of the minimum daily requirement. It is the manufacturer's intent to never average below the 20% level. A one-tail test would be applied in this case with the entire α risk on the left-tail.**

One-Tail Test (Continued)

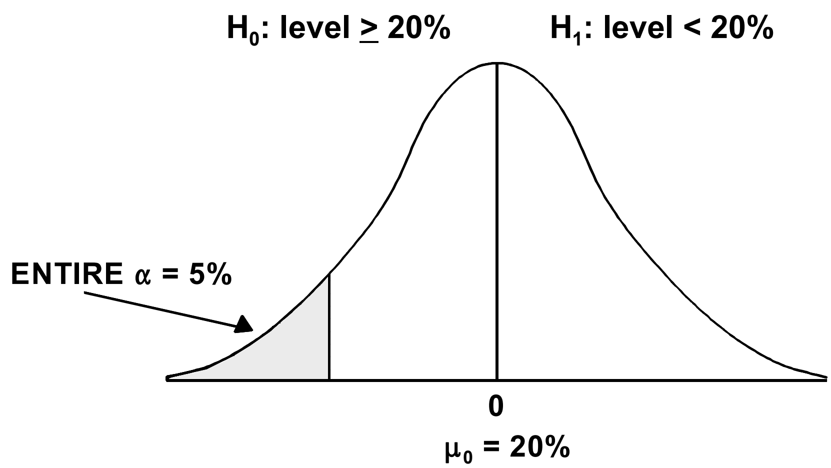


Figure 11.3 Determine if the true mean is within the α critical region.

Two-Tail Test

If a null hypothesis is established to test whether a population shift has occurred, in either direction, then a two-tail test is required. The allowable α error is generally divided into two equal parts. Examples:

- **An economist must determine if unemployment levels have changed significantly over the past year.**
- **A study is made to determine if the salary levels of company A differ significantly from those of company B.**

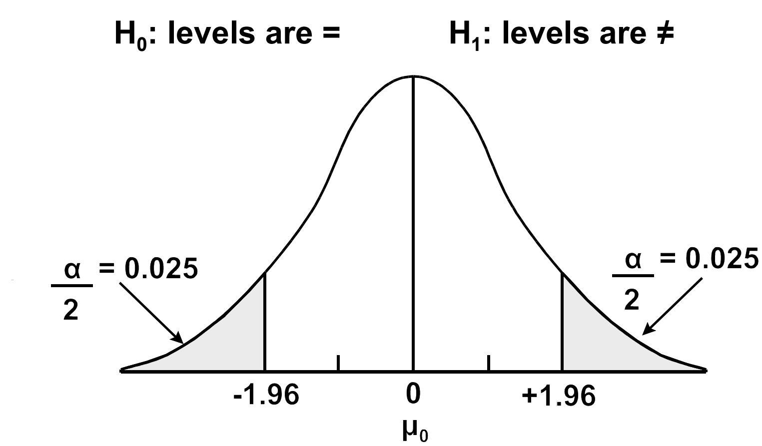


Figure 11.4 Determine if the true mean is within either the upper or lower α critical regions.

Required Sample Size

In the statistical discussion thus far, it has been assumed that the sample size, n, for hypothesis testing has been given and that the critical value of the test statistic will be determined based on the α error that can be tolerated. The ideal procedure, however, is to determine the α and β error desired and then to calculate the sample size necessary to obtain the desired decision confidence.

The sample size, n, needed for hypothesis testing depends on:

- The desired type I (α) risk and type II (β) risk
- The minimum value to be detected between the population means ($\mu - \mu_0$)
- The variation in the characteristic being measured (s or σ)

Variable data sample size, only using α, is illustrated by the following:

Example 11.6: Assume in a pilot process one wishes to determine whether an operational adjustment will alter the process hourly mean yield by as much as 4 tons per hour. What is the minimum sample size which, at the 95% confidence level (Z=1.96), would confirm the significance of a mean shift greater than 4 tons per hour? Historic information suggests that the standard deviation of the hourly output is 20 tons. The general sample size equation for variable data (normal distribution) is:

$$n = \frac{Z_{\alpha/2}^2 \sigma^2}{E^2} = \frac{(1.96)^2 (20)^2}{(4)^2} = 96$$

Obtain 96 pilot hourly yield values and determine the hourly average. If this mean deviates by more than 4 tons from the previous hourly average, a significant change at the 95% confidence level has occurred. If the sample mean deviates by less than 4 tons per hour, the observable mean shift can be explained by chance cause.

For binomial data, use the following formula:

$$n = \frac{Z_{\alpha/2}^2 (\bar{p})(1 - \bar{p})}{(\Delta p)^2}$$

Where:

Z = The appropriate Z value $\quad$ $\bar{p}$ = Proportion rate
Δp = The desired proportion interval $\quad$ n = Sample size

Hypothesis Tests for Means

Z Test

When the population follows a normal distribution and the population standard deviation, σ_X, is known, then the hypothesis tests for comparing a population mean, μ, with a fixed value, μ_0, are given by the following:

$H_0: \mu = \mu_0$	$H_0: \mu \leq \mu_0$	$H_0: \mu \geq \mu_0$
$H_1: \mu \neq \mu_0$	$H_1: \mu > \mu_0$	$H_1: \mu < \mu_0$

The null hypothesis is denoted by H_0 and the alternative hypothesis is denoted by H_1. The test statistic is given by:

$$Z = \frac{\bar{X} - \mu_0}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu_0}{\left(\frac{\sigma_X}{\sqrt{n}}\right)}$$

Where the sample average is $\bar{X}$, the number of samples is n and the standard deviation of means is $\sigma_{\bar{X}}$. If n > 30, the sample standard deviation, s, is often used as an estimate of the population standard deviation, σ_X. The test statistic, Z, is compared with a critical value, Z_α or $Z_{\alpha/2}$, which is based on a significance level, α for a one-tailed test or α/2 for a two-tailed test. If the H_1 sign is ≠, it is a two-tailed test. If the H_1 sign is >, it is a right, one-tailed test, and if the H_1 sign is <, it is a left, one-tailed test. (Triola, 1994)[9]

Example 11.7: The average vial height from an injection molding process has been 5.00" with a standard deviation of 0.12". An experiment is conducted using new material which yielded the following vial heights: 5.10", 4.90", 4.92", 4.87", 5.09", 4.89", 4.95", and 4.88".

Can one state, with 95% confidence, that the new material is producing shorter vials with the existing molding machine set-up? This question involves an inference about a population mean with a known sigma. The Z test applies. The null and alternative hypotheses are:

$H_0: \mu \geq \mu_0$ $\quad$ $H_1: \mu < \mu_0$

or

$H_0: \mu \geq 5.00"$ $\quad$ $H_1: \mu < 5.00"$

Hypothesis Tests for Means (Continued)

Z Test (Continued)

Example 11.7 continued:

The sample average is $\bar{X}$ = 4.95" with n = 8 and the population standard deviation is σ_X = 0.12". The test statistic is:

$$Z = \frac{\bar{X} - \mu_0}{\left(\frac{\sigma_X}{\sqrt{n}}\right)} = \frac{4.95 - 5.00}{\left(\frac{0.12}{\sqrt{8}}\right)} = -1.18$$

Since the H_1 sign is <, it is a left, one-tailed test and with a 95% confidence, the level of significance, α = 1 - 0.95 = 0.05. Looking up the critical value in a normal distribution or Z table, one finds $Z_{0.05}$ = -1.645. Since the test statistic, -1.18, does not fall in the reject (or critical) region, the null hypothesis cannot be rejected. There is insufficient evidence to conclude that the vials made with the new material are shorter.

If the test statistic had been, for example -1.85, one would have rejected the null hypothesis and concluded the vials made with the new material are shorter.

Student's t Test

The student's t distribution applies to samples drawn from a normally distributed population. It is used for making inferences about a population mean when the population variance σ^2 is unknown and the sample size n is small. The use of the t distribution is never wrong for any sample size. However, a sample size of 30 is normally the crossover point between the t and Z tests. The test statistic formula is:

$$t = \frac{\bar{X} - \mu_0}{\left(\frac{s_X}{\sqrt{n}}\right)}$$

Where: $\bar{X}$ **= The sample mean**
μ_0 **= The target value or population mean**
s_x **= The sample standard deviation**
n = The number of test samples

The null and alternative hypotheses are the same as were given for the Z test.

Hypothesis Tests for Means (Continued)

Student's t Test (Continued)

The test statistic, t, is compared with a critical value, t_α or $t_{\alpha/2}$, which is based on a significance level, α for a one-tailed test or α/2 for a two-tailed test, and the number of degrees of freedom, d.f. The degrees of freedom is determined by the number of samples, n, and is simply:

$$d.f. = n - 1$$

Example 11.8: The average daily yield of a chemical process has been 880 tons (μ = 880 tons). A new process has been evaluated for 25 days (n = 25) with a yield of 900 tons ($\bar{X}$) and sample standard deviation, s = 20 tons. Can one say, with 95% confidence, that the process has changed?

The null and alternative hypotheses are:

$$H_0: \mu = \mu_0 \qquad H_1: \mu \neq \mu_0$$

or

$$H_0: \mu = 880 \text{ tons} \qquad H_1: \mu \neq 880 \text{ tons}$$

The test statistic calculation is:

$$t = \frac{\bar{X} - \mu_0}{\left(\frac{s_X}{\sqrt{n}}\right)} = \frac{900 - 880}{\left(\frac{20}{\sqrt{25}}\right)} = 5.0$$

Since the H_1 sign is ≠, it is a two-tailed test and with a 95% confidence, the level of significance, α = 1 - 0.95 = 0.05. Since it is a two-tailed test, α/2 is used to determine the critical values. The degrees of freedom, d.f. = n - 1 = 24. Looking up the critical values in a t distribution table, yields $t_{0.025}$ = -2.064 and $t_{0.975}$ = 2.064. Since the test statistic, 5, falls in the right-hand reject (or critical) region, the null hypothesis is rejected. One concludes, with 95% confidence, that the process has changed.

This technique was developed by W.S. Gosset and published in 1908 under the pen name "Student." Gosset referred to the quantity under study as t. The test has since been known as the student's t test.

Hypothesis Tests for Means (Continued)

Student's t Test (Continued)

Example 11.9: A new spark plug design is tested for wear. A sample of six plugs yielded: 0.0058", 0.0049", 0.0052", 0.0044", 0.0050" and 0.0047", of wear. The current design has historically produced an average wear of 0.0055". With 95 % confidence, is the new design better?

Step 1: Establish the null hypothesis: The new spark plug has equal to or greater than the historical average wear. The alternative hypothesis: The new spark plug has less than the historical average wear.

$$H_0: \mu \geq \mu_0 \quad \text{or} \quad \mu \geq 0.0055"$$
$$H_1: \mu < \mu_0 \quad \text{or} \quad \mu < 0.0055"$$

Step 2: Determine the critical value of t for a 95% confidence level from the t distribution, with d.f. = n - 1 = 6 - 1 = 5. For left tail, the critical t = -2.015.

Step 3: Calculate the sample mean and sample standard deviation. This can be done using the formulas previously given or on most scientific calculators:

$$\bar{X} = 0.0050" \qquad s_X = 0.00048"$$

Step 4: Calculate the t statistic:

$$t = \frac{\bar{X} - \mu_0}{\left(\frac{s_X}{\sqrt{n}}\right)} = \frac{0.0050 - 0.0055}{\left(\frac{0.00048}{\sqrt{6}}\right)} = -2.551$$

Step 5: Can the null hypothesis be rejected? Since the value of calculated t is to the left of -2.015, and in the critical area, the null hypothesis is rejected. The wear is less for the new plug design.

Hypothesis Tests for Means (Continued)

Student's t Test (Continued)

Example 11.10: A very expensive experiment has been conducted to evaluate the manufacture of synthetic diamonds by a new technique. Five diamonds have been generated with recorded weights of 0.46, 0.61, 0.52, 0.57 and 0.54 carats. An average diamond weight greater than 0.50 carats must be realized for the venture to be profitable, assuming 95% confidence, what is the recommendation?

Step 1: Establish the null hypothesis: The new technique produces equal to or less than the average diamond weight of 0.50 carats. The alternative hypothesis: The new technique produces an average diamond weight of more than 0.50 carats.

$$H_0: \mu \leq \mu_0 \quad \text{or} \quad \mu \leq 0.50 \text{ carats}$$
$$H_1: \mu > \mu_0 \quad \text{or} \quad \mu > 0.50 \text{ carats}$$

Step 2: Determine the critical value of t for a 95% confidence level from the t distribution, with d.f. = n - 1 = 5 - 1 = 4. For right-tail, the critical t = 2.132.

Step 3: Calculate the sample mean and sample standard deviation. This can be done using the formulas previously given or on most scientific calculators:

$$\bar{X} = 0.54 \text{ carats} \qquad s_X = 0.056 \text{ carats}$$

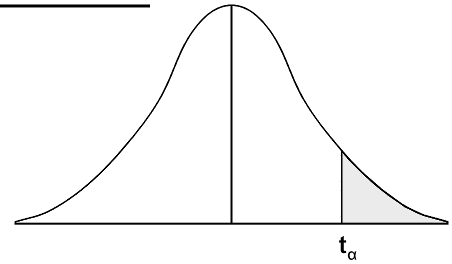
Step 4: Calculate the t statistic:

$$t = \frac{\bar{X} - \mu_0}{\left(\frac{s_X}{\sqrt{n}}\right)} = \frac{0.54 - 0.50}{\left(\frac{0.056}{\sqrt{5}}\right)} = 1.597$$

Step 5: Can the null hypothesis be rejected? Since the value of calculated t is not to the right of 2.132, it is not in the critical area, and the null hypothesis cannot be rejected. Insufficient evidence exists for the new technique to be profitable.

One underlying assumption is that the sampled population has a normal probability distribution. This is a restrictive assumption since the distribution of the sample is unknown. The t distribution works well, however, for distributions that are bell shaped.

t Distribution Table



d.f.	$t_{0.100}$	$t_{0.050}$*	$t_{0.025}$**	$t_{0.010}$	$t_{0.005}$	d.f.
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
inf.	1.282	1.645	1.960	2.326	2.576	inf.

* One-tail 5% α risk ** Two-tail 5% α risk

Table 11.5 Student's t Distribution Table

There is only a 5% probability that a sample with 10 degrees of freedom will have a t value greater than 1.812.

Hypothesis Tests for Proportions

p Test

When testing a claim about a population proportion and we have a fixed number of independent trials with constant probabilities, and each trial has two outcome possibilities (a binomial experiment), we may use a p test. When $np < 5$ or $n(1-p) < 5$, the binomial distribution is used to test hypotheses relating to proportion.

If conditions that $np \geq 5$ and $n(1-p) \geq 5$ are met, then the binomial distribution of sample proportions can be approximated by a normal distribution. The hypothesis tests for comparing a sample proportion, p, with a fixed value, p_0, are given by the following:

$H_0: p = p_0$	$H_0: p \leq p_0$	$H_0: p \geq p_0$
$H_1: p \neq p_0$	$H_1: p > p_0$	$H_1: p < p_0$

The null hypothesis is denoted by H_0 and the alternative hypothesis is denoted by H_1. The test statistic is given by:

$$Z = \frac{x - np_0}{\sqrt{np_0(1 - p_0)}}$$

Where the number of successes is x and the number of samples is n. The test statistic, Z, is compared with a critical value, Z_α or $Z_{\alpha/2}$, which is based on a significance level, α, for a one-tailed test or α/2 for a two-tailed test. If the H_1 sign is ≠, it is a two-tailed test. If the H_1 sign is >, it is a right, one-tailed test, and if the H_1 sign is <, it is a left, one-tailed test. (Triola, 1994)[9]

